Microwave Noise in Semiconductor Devices

Module 3

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1 Physical aspects of noise and inclusion in linear networks

[Ambrozy Chaps 4,5]

Due to time will not go into great detail about subtleties of shot noise. Most of the time the original formula we derived works fine. Key points:

1.1 Shot noise

1. If , corrections are needed to the shot noise formula. This correction depends on the geometry of the system exhibiting shot noise (planar, cylindrical, etc). Denote this correction .

2. As mentioned in the last Module, the shot noise spectrum that is measured is typically modified by the network's transfer function. For a single pole RC circuit, and including the geometrical correction, the shot noise power spectrum is:

Typically the RC roll-off dominates the other effect.

1.2 Partition noise

In some cases charge carriers follow random paths and end up at different detectors. There is a noise associated with this process.

We will have a homework problem to derive the following result:

Note that if , we get the original shot noise power spectrum back.

1.3 Generation-recombination noise

This noise source arises due to the thermal generation and recombination of electrons and holes in trap states. It is usually small compared to other noise sources in modern devices. Further, time scale associated with the process is order microseconds so that the noise is well below the frequency bands of interest to us.

However, intervalley noise can be thought of as a type of GR noise and so it is of interest. You will have a homework problem to derive the spectral noise power from this mechanism. Here is the general idea. The resistance of a homogeneous semiconductor bar is

where is the length and is the cross-sectional area.

The number densities of electrons and holes are , their respective mobilities, and the total numbers are .

The number densities may fluctuate due to generation and recombination (or intervalley transfer to another valley). Specifically focusing on traditional GR, we have both electron and hole densities changing as

Therefore, the resistance of the bar fluctuates stochastically:

Using these facts, one can derive the power spectrum of this noise mechanism to be:

1.4 Flicker noise

This noise source is observed in thin metallic or semiconductor layers (carbon resistors, solid state devices, etc). It has a PSD as:

For , the noise is associated with a fluctuation in resistance. For (the famous 1/f noise), the physical origin is still murky. Again, this noise is not relevant in our frequency band.

1.5 Thermal noise

It was observed in the late 1920s that homogeneous semiconductors in equilibrium at a given temperature exhibited voltage and current noise, and that this noise varied with temperature. (In fact, this Johnson-Nyquist noise can be used as a primary thermometer.)

I will present several derivations of the thermal noise PSD over the course of the class. Each method highlights a different way to go about analyzing noise based on our mathematical foundation.

Nyquist derivation [Nyquist 1928] Say we have a noisy resistor and a noise-less resistor, both of resistance , connected by a perfect transmission line. Let's model the noisy resistor as a noise-less resistor + fluctuating current source to get the following circuit.

By standard EM theory, fluctuating currents produce electromagnetic waves. In the present case, these waves travel down the modes of a 1D transmission line. We want to calculate the power radiated by the noisy resistor.

We need two ingredients: the density of states of EM modes in a 1D transmission line, and the energy per mode.

First, DOS for a 1D transmission line. The modes can be thought of as normal modes so that we use the standard approach from solid-state physics:

Next, we need the energy per mode. Let's take the classical limit for now. By equipartition the answer is just . Therefore, the electromagnetic energy on the line per unit bandwidth is thus

Half of energy is radiated from the source to the load in time , where it is completely absorbed as the impedances are the same. The power transfer is

On the other hand, from basic circuits the power dissipated in the second resistor is:

Putting these two together, we get:

which is the famous Nyquist formula.

We can get the spectral density of current fluctuations as

This derivation requires no knowledge of microscopic quantities, just macroscopic impedances.

2 Noise parameters of linear networks

With the basic expressions for noise power from different physical mechanisms derived, we can use them to construct noise-equivalent circuits with noise current or voltage generators.

2.1 One-ports

A two pole circuit with conductance looks like:

We know that this conductance should emit a noise current (or noise voltage, equivalently). We can make either one happen with

Noise current generator in parallel Noise voltage generator in series

[We should not be surprised that these two representations are the same due to the source transformation theorem of circuit theory.]

Example: confirm that the noise power delivered to load for the voltage circuit is indeed .

Answer: Start from source current of current generator:

Perform a Norton-Thevenin transformation

The voltage dropped across the matched load is Therefore the power delivered is

Note that the source parameters of the generators include the bandwidth. While inconvenient, often we care about a narrow frequency band so that we use:

These noise equivalent circuits can be applied with networks containing reactive elements too. Taking the reactive component as ideal so that it does not generate noise, we simply take into account the admittance of the element.

Example: put a capacitor in parallel with the conductance. Then

This result also follows from our previous derivation that the output noise power density is related to the input power density by the transfer function as:

This analysis works for a single noise source. What is two or more correlated noise sources exist in a network? In this case, we need to consider cross correlation between the noise generators.

Let's consider an example with two noise voltage generators. Take to be dependent stationary stochastic processes. The sum variable is

The autocorrelation of is

The first and last terms are ACF of , two middle ones are cross-correlation functions (CCF). These latter two are not independent. To see that, consider:

Considering that the processes are stationary, we get:

Since the original expressions were equal, the middle terms must be equal so that:

Thus the CCF is neither even nor odd.

From the ACF and CCF we can calculate the power spectrum. The definition is:

For the ACF, we know they are even so that:

The CCF are not even so we need to use the original definition. We get:

Finally, we have:

If the two stochastic processes are independent, then

If we confine ourselves to a narrow frequency band ,

Dimensionally, the RHS terms have units of and therefore can be interpreted as powers of the stochastic processes in frequency band

[Technically we have the power for a load].

Call these powers .

Now consider the cross-terms. We previously derived that for a small enough bandwidth a stochastic signal can be approximated by a sine (or cos) signal of frequency . In particular,

if $\Delta\omega \ll \omega_0$ and $\tau\Delta\omega \ll 2\pi$.

Take the complex amplitude of the sinusoidal frequency to be and and use our previous result of the ACF of a sinusoidal function. Then,

We then see that the mean square voltage fluctuation is:

This expression can be generalized to several voltage and/or current generators. The result is that the narrow-band overall noise voltage (or noise current) is

Example:

While this result is technically valid only for a narrow frequency band, we can perform the calculation at various frequencies and superpose the results for a linear network. The frequency can only affect the coefficients of the noise generators.

2.2 Two-ports

Consider a matrix equation of the type

It gives a relationship between source voltages and currents of a linear, lumped parameter two-port element.

The independent variables are , dependent variables are , is the network matrix, and characterize the inhomogeneity.

Depending on whether current and/or voltage are taken as independent or dependent, we can get six types of matrix equations.

Example: admittance equation

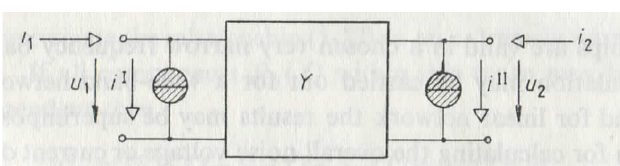

Figure 5-4 Two-port with noise sources excluded.

Figure 1:

If the network has only controlled generators, . So the inhomogeneous terms originate from independent generators.

In an electronic device, DC supply power is converted into the output AC signal according to the input driving signal. So in a small-signal, noise-free equivalent circuit, there are no independent generators (the effect is lumped into the two-port element).

To include noise, we can put noise sources at different sites of the passive or active network. Since the circuit is linear, the noise voltage or current depends only on location, not driving conditions.

Here we find a difference between one-ports and two-ports.

One-port: requires only a single parameter to characterize (cf resistance).

Two-port: requires two uncontrolled generators that may not be independent. We need a total of 4 data points to characterize them (one for each generator, two for complex correlation between them).

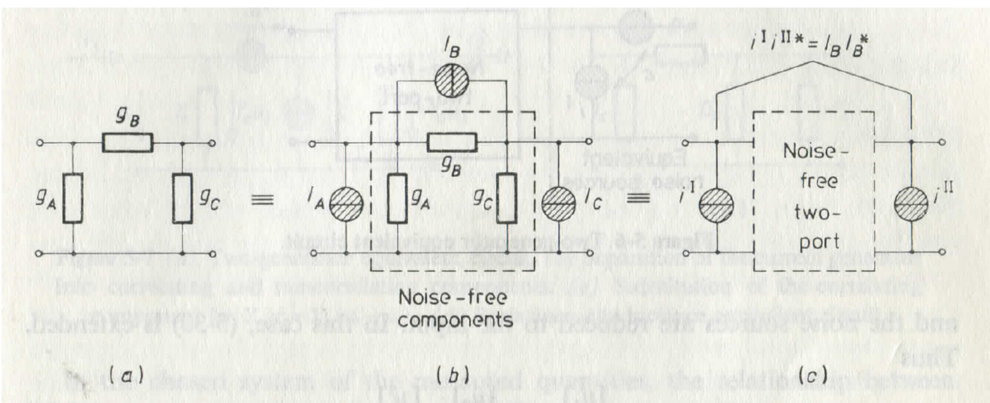

Figure 5-5 Two-port noise equivalent circuits. *(a)* Noisy two-port. *(b)* Exclusion of noise sources. *(c)* Contraction of noise sources.

Figure 2:

Example: consider the noise equivalent circuit of (a) above. The thermal noise currents are

To get the source current of the input equivalent noise generator, we short the output and connect an ideal current meter of zero resistance to the input.

[Because if , referring to admittance equations]

To get the overall noise, use the noise addition formula:

Similarly,

The correlation is characterized by:

which in this case is real. So now we can regard the block in (c) as noise-free, with internal noise sources substituted by and their correlation.

We can refer the noise sources back to input with another two-port representation. We use the representation for the noiseless two-port:

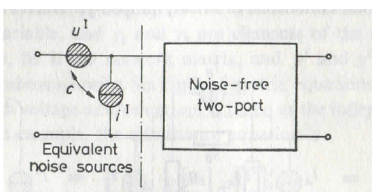

Figure 5-6 Two-generator equivalent circuit.

Figure 3:

The noise sources are now at the input; we account for that in the matrix equations as

To complete the description, we need to specify the correlation factor

A drawback of this approach is that the bandwidth is in the source parameters. To avoid this, we can make an equivalent circuit based on equivalent noise resistance conductance, and correlation admittance.

Consider the two-port generator below. Denote input and output quantities by subscripts

and , respectively. Lower case indicates instantaneous values.

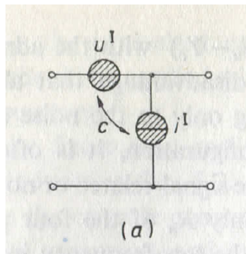

Figure 4:

Separate the noise current into two components.

Perfect correlation with :

Independent component:

Perfect correlation means that:

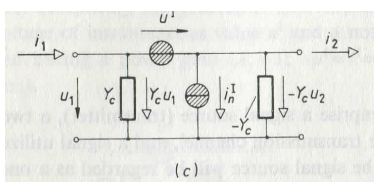

Figure 5:

where is the correlation admittance.

Therefore, the relation between input and output two-port currents and voltages is

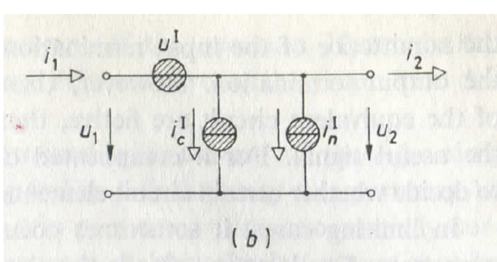

Figure 6:

We therefore get:

Note that and are independent.

We can now introduce equivalent noise resistance and conductance :

yielding the following equivalent circuit:

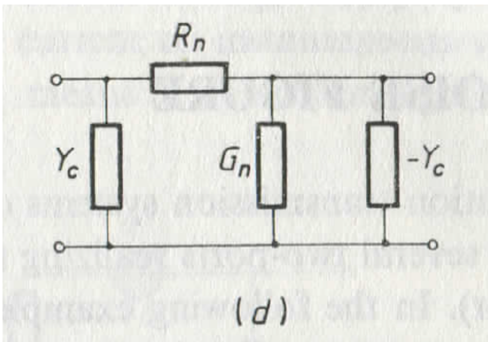

Figure 7:

The parameters of this circuit do not contain the bandwidth (although they will in general depend on frequency). We can further simplify by contracting with the admittance of the input termination, and similarly with the admittance of output termination.

A disadvantage is that these circuit elements are fictitious (they describe noise, not the real signal). Complications can arise in complex circuits to determine if various circuit elements are related to the signal or noise.

2.3 Noise figure

A generic signal transmission system consists of a transmitter, several two-ports consisting of the transmission channel, and a receiver. Take the signal source to be a one-port.

If a two-port has a power gain , then times the signal source noise power appears at the output, plus the noise power generated within the two-port.

We use the noise figure to characterize the ratio of these noise powers of different origins:

Here is the total output noise power, is the fraction of this power generated within the two-port.

Noise figure can be defined only for a signal source with a finite resistance so that .

We observe that since at best (neglecting quantum constraints) the two-port adds zero noise power.

We can convert the noise figure to decibels:

If the two-port added noise is sufficiently small, we use noise temperature. Referring back to input:

For a matched case we can write

Since the output noise power referred to input is bigger than , we can write

which defines the equivalent noise temperature .

We have to be careful to check that all parts of the circuit have the same temperature in applying this result.

Let's see how to compute the noise figure in a cascaded two-port.

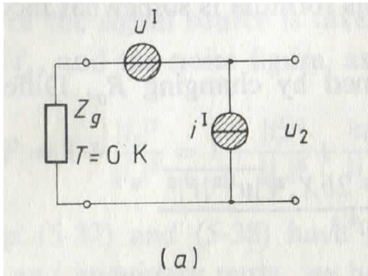

Figure 8:

Consider this circuit. The circuit has a noiseless generator impedance , supplies a noise voltage and noise current (instantaneous values) with power gain

To calculate the noise figure, let's get the squared absolute value of the open circuit voltage. From our earlier derivation we have

Take the generator impedance to be
correlation factor

We then have

However, we know that the generator can create a noise voltage:
It is uncorrelated so the magnitude of this noise source and original one add.
The noise figure is then

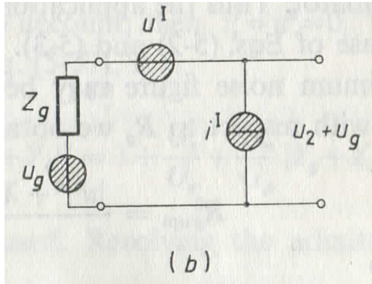

Figure 9:

For a noise-free load resistance connected to the output terminals of the two-port, the ratio of remains unchanged. Therefore, we get the noise figure

$$F = 1 + \frac{|u^I|^2 + R_g^2|i^I|^2 + X_g^2|i^I|^2 + 2\sqrt{|u^I|^2|i^I|^2}(aR_g - bX_g)}{4k_BTR_g\Delta f} \tag{1}$$

The equation tells us that the four noise parameters ( ) of the two-port, and input termination values allows us to compute the noise figure. The value is defined for a bandwidth .

We can get the minimum noise figure by varying . The optimal value is that which minimizes :

$$R_{g,opt}^2 = \frac{|u^I|^2 + X_g^2|i^I|^2 - 2bX_g\sqrt{|u^I|^2|i^I|^2}}{|i^I|^2} \tag{2}$$

If :

which is independent of the correlation factor .

If but , then the optimal source resistance is:

Now, since we already showed that the two-generator and noise resistance-conductance equivalent circuits and themselves equivalent, we can express the noise figure in terms of either one.

Consider this circuit.

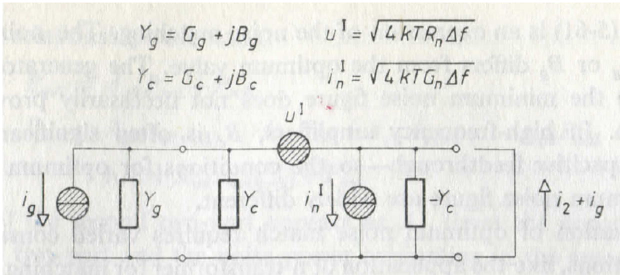

Figure 5-9 Noise figure calculation from the equivalent circuit shown in Fig. 5-7*c*.

Figure 10:

The short-circuit output current for is

since are independent. If we only account for the signal noise source, then and the output current is . So the noise figure is:

Finally, include the real and imaginary parts of the admittances to get:

$$F = 1 + \frac{G_n}{G_g} + \frac{R_n}{G_g}\left[(G_n + G_c)^2 + (B_n + B_c)^2\right] \tag{3}$$

If , or if , then

The minimum value of the noise figure is obtained with

so that

If we take , we get:

$$F - F_{min} = \frac{R_n}{G_g}\left[(G_g - G_{g,opt})^2 + (B_g - B_{g,opt})^2\right] \tag{4}$$

These equations describe the concept of noise-matching. If the generator admittance differs from its optimum value, the noise increases.

Note that the optimal noise admittance is not necessary giving the best power match (or gain match). So, the conditions for optimum power transfer and minimum noise are generally quite different.