Microwave Noise in Semiconductor Devices

Module 6

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1 Fluctuation-dissipation in equilibrium and non-equilibrium

[Kogan Chaps 2,3]

Our goal is to eventually be able to understand the microscopic origin of current noise in an electron gas driven by an external field. Before we do that, we need to treat the equilibrium situation more rigorously.

The fluctuation-dissipation theorem is a generalized version of the original Nyquist theorem. It relates the spectral noise power to the dissipative part of the linear response of a system to an external field. It was originally derived in its general form by Callen, Welton, Green, Barasch, and Jackson in a series of papers in 1951-2.

1.1 Fluctuation-dissipation theorem

[Callen-Welton approach]

Consider a system in equilibrium with a thermal bath at temperature T. From stat mech, the probability to find the system in state m with energy E_m is the Gibbs factor:

$$w_m = \frac{e^{-E_m/k_bT}}{Z} = e^{(F-E_m)/k_bT}$$

$$(F = \text{free energy} = -k_bT \ln Z)$$

With these probabilities we can define a density matrix using the eigenstates of the Hamiltonian:

$$|\psi_m\rangle = \text{eigenstates}$$

$$\rho = \sum_m w_m |\psi_m\rangle\langle\psi_m|$$

This density matrix is diagonal in this basis with eigenvalues W_m .

We want to calculate the spectral density of fluctuations and the linear response of the system to a perturbation. By comparing these results, we will see they are related.

Let's tackle the spectral density of fluctuations for some quantity $x(t)$ in an equilibrium system. In quantum mechanics, an observable is an operator, so that $x(t) \to \hat{x}(t)$. Further, we want the correlation of this operator with itself at a different time. This quantity is measurable and so must be Hermitian. However, the classical correlation function is not Hermitian. We fix this by defining the quantum correlation function as

$$\frac{1}{2}\left\{ \hat{x}(t_1), \hat{x}(t_2) \right\} = \frac{1}{2}\left(\hat{x}(t_1)\hat{x}(t_2) + \hat{x}(t_2)\hat{x}(t_1) \right)$$

Now we can write:

$$S_x(f) = 2\int_{-\infty}^{\infty} d(t_1 - t_2) \cdot \frac{1}{2} Tr\left(\rho \left\{ \hat{x}(t_1), \hat{x}(t_2) \right\} \right) e^{j\omega(t_1 - t_2)}$$

thermal average

$$X_{mn} = \langle \psi_m | \hat{x} | \psi_n \rangle$$

We write $X_{mn}(t)$ as the matrix element of the Heisenberg operator $\hat{x}(t)$ between states ψ_m, ψ_n. This operator follows the Heisenberg equation of motion (recall the Heisenberg picture is the one in which the operators have time-dependence, not the state).

$$\hat{x}(t) = U^{\dagger} \hat{x} U \quad , \quad U = e^{-jHt/\hbar}$$

$$\frac{\partial \hat{x}}{\partial t} = \frac{jH}{\hbar}\underbrace{(U^{\dagger}\hat{x}U)}_{\hat{x}(t)} + \frac{U^{\dagger}\hat{x}(-jH)U}{\hbar} = (jH\hat{x}(t) - j\hat{x}(t)H)/\hbar$$

$$= \frac{j}{\hbar}[H, \hat{x}(t)]$$

Apply $\langle \psi_m |, |\psi_n\rangle$ to each side:

$$\frac{\partial X_{mn}(t)}{\partial t} = \frac{j}{\hbar}\langle \psi_m | [H, \hat{x}] | \psi_n \rangle = \frac{j}{\hbar}\left(E_m X_{mn} - E_n X_{mn} \right)$$

$$= \frac{j}{\hbar}(E_m - E_n) X_{mn}$$

$$= j\omega_{mn} X_{mn}$$

This equation has solution:

$$X_{mn}(t) = X_{mn}(0) \, e^{j\omega_{mn}t} \quad \longleftarrow$$

Use the definition of delta function to get:

$$\delta(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} \, dt$$

$$T_r \equiv \sum_m \langle \psi_n | \cdot | \psi_m \rangle$$

$$I = \sum_n |\psi_n\rangle\langle\psi_n|$$

$$\boxed{S_x(f) = 2\pi \sum_{mn} W_m |X_{mn}|^2 \left[\delta(\omega_{nm}-\omega) + \delta(\omega_{nm}+\omega) \right]}$$

[This step takes a few lines of derivation and will be a homework problem.]

We see that the spectral density of fluctuations of $x(t)$ at frequency f are determined by the matrix elements of the QM operator $\hat{x}$ between states with energies differing by $E_m - E_n = \pm\, hf$.

Simplify a bit more by interchanging m and n in the second sum to get:

$$S_x(f) = 2\pi \sum_{mn} (W_m + W_n) |X_{mn}|^2 \, \delta(\omega_{nm}-\omega)$$

Now remember that we know the weights for the equilibrium system as the Gibbs weights.

In particular, we know

$$\frac{W_n}{W_m} = e^{-\hbar\omega_{nm}/k_B T}$$

So the final expression for spectral density of fluctuations is:

$$\curvearrowright \quad S_x(f) = 2\pi \left[1 + e^{-hf/k_B T} \right] \sum_{mn} W_m |X_{mn}|^2 \, \delta(\omega_{nm}-\omega)$$

Now consider an apparently unrelated problem of the response of a system to an external perturbation. Say our physical variable x interacts with an externally applied field F . $\leftarrow$ not free energy

If $\hat{x}(t)$ is the QM operator, the Hamiltonian interaction term can be written $H = -F\hat{x}$.

Example: say p_x is a component of a dipole moment $\bar{P}$ and $\bar{F}$ is the electric

field acting on the system. The Hamiltonian term is $H = -P_x F_x$.

Express $F(t)$ as a Fourier series and examine its effect at angular frequency ω ($\omega = 2\pi f$). The

driving field $F(t)$ is a real function so that:

$$F(t) = \frac{1}{2}\left(F_0\, e^{-j\omega t} + F_0^*\, e^{j\omega t}\right)$$

This external field forces the system to undergo transitions between its eigenstates and

ultimately dissipate the energy. From basic perturbation theory in QM, we have Fermi's

golden rule for the transition rate due to an external field:

$$\longrightarrow \quad W_{mn} = \frac{\pi}{2\hbar^2}\, |F_0|^2\, |x_{mn}|^2\left[\,\delta(\omega_{nm} - \omega) + \delta(\omega_{mn} - \omega)\right]$$

When a transition happens, an energy quantum $\hbar\omega$ is either absorbed or emitted. The

average rate of power transfer is

$$\bar{Q} = \sum_{mn} W_m \overset{\text{transition rate}}{\overset{\downarrow}{W_{mn}}} (\hbar\omega_{nm})$$

$$\underset{\text{Gibbs's factor}}{mn}$$

With this transition rate we can compute the power transferred to the system by averaging

over the initial Gibbs state:

$$\longrightarrow \quad \bar{Q} = \frac{\pi\overset{\downarrow}{\omega}}{2\hbar}\, |F_0|^2\left(1 - \underset{\text{Gibbs's factor}}{e^{-\hbar f/k_B T}}\right)\sum_{mn} W_m\, |x_{mn}|^2\, \delta(\omega_{nm} - \omega)$$

This looks something like $S_x(f)$! Let's figure out the prefactor.

Consider the most general expression for the linear response of X to an external

force F :

$$\langle x(t)\rangle = \int_{-\infty}^{t} dt'\, A(t-t')\, F(t')$$

$$\overset{\text{causality}}{\ell} \qquad \qquad \underset{\text{response}}{\ell}$$

[We have seen this equation before.] Expanding $F(t)$ in a Fourier series again, we get

$$\langle x(t)\rangle = \frac{1}{2}\left[F_0\, A(\omega)\, e^{-j\omega t} + F_0^*\, A(-\omega)\, e^{j\omega t}\right]$$

where $\quad A(\omega) \;=\; \displaystyle\int_0^\infty dt\; e^{j\omega t}\, A(t) = A^*(-\omega)$

is the frequency-dependent susceptibility of the system. The power absorbed by the system and dissipated over period $2\pi/\omega$ is:

$$\bar{Q} = \langle \dot{x}(t) \rangle \cdot F(t)$$

$$Q = \frac{1}{4}\left[-j\omega\, F_0\, A(\omega)\, e^{-j\omega t} + j\omega\, F_0^*\, A^*(\omega)\, e^{j\omega t} \right] \times$$

$$\left[F_0\, e^{-j\omega t} + F_0^*\, e^{j\omega t} \right]$$

Only the terms lacking time dependence contribute to the average power. So

$$\bar{Q} = \frac{1}{4}\, |F_0|^2\, j\omega\, \left(A^*(\omega) - A(\omega) \right)$$

$$= \frac{1}{2}\, |F_0|^2\, \omega\, A''(\omega)$$

where $\quad A''(\omega) \equiv \mathrm{Im}(A) \quad$ is the imaginary part of the complex susceptibility.

Using this and the previous equation for power dissipation, we get:

$$\left[S_x(f) = 2\hbar\, \frac{(1 + e^{-hf/k_s T})}{(1 - e^{-hf/k_s T})}\, A''(\omega) = 2\hbar\, \coth\!\left(\frac{hf}{2k_s T}\right) A''(\omega) \right]$$

This is the fluctuation-dissipation relation. It links the spectral density of fluctuations to the lossy part of the linear response of the system. To get it in its final form, we usually
A''
write it in terms of a generalized conductance, defined as the response of $\dot{x}$ to $F(t)$.

Considering the various definitions, we have:

$$G(\omega) = -j\omega\, A(\omega), \quad G' \equiv \mathrm{Re}\, G = \omega\, A''$$

$$S_{\dot{x}}(f) = \omega^2\, S_x(f)$$

$$S_{\dot{x}}(f) = 2hf\, \coth\!\left(\frac{hf}{2k_s T}\right) G'(\omega)$$

Finally, let's apply it to an Ohmic conductor with impedance Z . In this case the external field is a voltage, and the time-dependent response is a current. So we want the spectral intensity of current fluctuations. Since $G = Z^{-1}$, we get:

$$S_I(f) = 2hf \coth\left(\frac{hf}{2k_bT}\right) \text{Re}\left(\frac{1}{Z}\right) \quad \leftarrow$$

Calling $\text{Re}(Z^{-1}) \to R^{-1}$ and taking the limit of $hf \ll k_bT$, we get

$$S_I(f) = 4k_bT\,R^{-1} \quad \leftarrow \text{ usually good}$$
$$\text{for microwave applications}$$

which is what we have gotten from the original Nyquist derivation. Note that the term incorporating temperature and frequency depending can be written

$$\frac{hf}{2}\coth\left(\frac{hf}{2k_bT}\right) = hf\left[\bar{N}(f,T) + \frac{1}{2}\right]$$

where

$$\bar{N} = \frac{1}{e^{hf/k_bT} - 1}$$

is the mean number of quanta of an oscillator with frequency f , temperature T .

Some numbers: at $T = 1\ K$, $\frac{hf}{2k_bT} > 1$, for $f > 40\,GHz$.

2   Fluctuations in non-equilibrium gases

With the mathematical machinery in place, we now return to the original problem: understanding the noise observed in an electron gas with a field applied. Or, how drain noise can have an equivalent noise temperature of >1000 K, much greater than the physical temperature?

In equilibrium, obtaining the spectral noise power of a solid-state plasma (free electron gas in a solid-state host) is easy - measure the resistance. FDR then gives the noise power, which has a white spectrum for nearly all frequencies of interest.

For a non-equilibrium plasma driven by an external field, we lack a general relation between spectral density and response properties. So we need to further develop the theory.

We will now figure out to do that calculation for a gas of hot electrons that are not equilibrium with the phonon thermal bath of the conductor. We neglect pair collisions of electrons (considering electron-phonon collisions to dominate).

2.1 Some history

- 1935 - Leontovich first investigated how to treat fluctuations around stationary, non-equilibrium states, calculated correlations for monomolecular (one particle changes state) and bimolecular (two particles simultaneously change state. In this case a correlation arises between particles).

- 1953 - Wannier uses the BE *[↙ Boltzmann equation]* to describe random motion of ions about a mean drift distribution. He introduced a generalized Einstein relation that is true for systems with a constant relaxation time but otherwise not valid.

- 1956 - Hashitsume gives a statistical theory of linear dissipative systems using Langevin type analysis.

- 1957 - Kadomtzev describes BE with Langevin sources → Boltzmann Langevin equation.

- 1959 - Price reports early theory of hot electron noise, showing (1) spectral noise power can be related to a differential diffusion coefficient, and (2) predicting anisotropy due to 'energy relaxation noise' although he does not call it that. It is the second term in Eq 19.

- 1960 - 66— Lax develops a theory of noise for monomolecular transitions in a series of papers that treat fluctuations from a non-equilibrium steady state. His work treats the same problem as Wannier with a more general mathematical treatment.

- 1962 - Erlbach & Gunn measure transverse noise temperature of hot electrons in Ge at 420 MHz, observing an increase in T_n with E field (presumably due to heating of the gas). The expected anisotropy is not apparently observed although it is hard to tell.

- 1963 - Kogan analyzes electronic noise that is induced by temperature fluctuations of the electron gas in a macroscopic approach, assuming that electrons possess a well-defined temperature.

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- 1963 - Gurevich uses a Boltzmann-kinetic type treatment to analyze current noise in semiconductors. It is an early version of the "method of moments" in which a Boltzmann-type equation is derived for the current autocorrelation function. There is an error in this paper. Also in this year, Gurevich describes what is known as convective noise, noting that it will exhibit a frequency dependence at f and anisotropy according to E field axis.

- 1966 - Gurevich Katilius analyze electron transport in an anisotropic semiconductor. The treatment closely follows the 1963 paper and corrects an error in that paper (around eq 3.8, fixing eq 6 in earlier paper). This paper appears to correctly include correlation owing to the fixed number of particles.

- 1967 - Kogan & Shulman: computation of noise spectrum of hot electrons with frequent ee collisions from a macroscopic perspective (Sec 3.1 of book).

- 1967-8 - Bareikis, Matulioniene, Pozela report measurements of anisotropic noise temperature in p-type Ge, T = 80 K, 9.6 GHz, confirming prediction of 'convective' noise and anisotropy.

- 1968 - Levinson & Matulis examine theoretically noise including inelastic scattering from optical phonons. Equation for equal-time correlator.

- 1968 - Ruch and Kino: measurements of mobility, diffusion coefficient, etc in GaAs.

- 1969 - Bixon & Zwanzig - Described Boltzmann Langevin equation and showed how to compute fluctuations of pressure and heat current.

- 1969 - Kogan & Shulman; Langevin approach to computing current noise power with Green's functions. Kogan's work corresponds to Chap 3 of book. Gantsevich, Gurevich, Katilius: rigorous derivation of "method of moments" approach to noise from quantum statistical mechanics.

- 1969 - Sigmon & Gibbons, measurement of longitudinal high-field diffusion of electrons and holes in silicon.

- 1970 - Fox & Uhlenbeck: Theory of hydrodynamical fluctuations.

- 1970 - Kogan & Shulman, Gantsevich, Gurevich, Katilius (1971, JETP): equivalence of various methods to compute spectral density of nonequilibrium gas. Gantsevich, Gurevich, Katilius (another paper) studies scattering of light by hot electrons using a diagrammatic method; it now includes spatial inhomogeneity to the theory.

- 1970 - Shulman & Kogan: spatially inhomogeneous fluctuations in a non-equilibrium gas.

- 1970 - Bartelink & Persky: measurement of transverse diffusion coefficient in Si at 300 K.

- 1971 - van Vliet: Markov approach for density fluctuations, showing effect of correlation of non equilibrium gas with pair collisions.

- 1972 - Holm-Kennedy & Champlin: measured warm electron microwave dielectric constant and resistivity in Si. Humps are observed, attributed to intervalley processes.

- 1974 - Van Kampen: equation for fluctuations in a homogeneous dilute gas using only assumptions of Boltzmann equation.

- 1975 - V. Barekeis, V. Viktoravichyus, A. Gal'dikas, and R. Milyushite: Measurements of anisotropy of noise in n-type Ge at microwave frequencies.

- 1976 - Logan & Kac: describe evolution of homogeneous gas by a Markov process obtaining Fokker-Planck type equations.

- 1978 - Price surveys numerical methods for hot electron noise.

- 1978 - Onuki: master equation to derive fluctuations about a mean distribution in an inhomogeneous gas. Nonequilibrium correlations lead to non-exponential decay of response.

- 1978 - V. Barekeis, V. Viktoravichyus, A. Gal'dikas, and R. Milyushite: Measurements of electronic noise, diffusion coefficient, and small-signal conductivity in Si and Ge.

- 1978 - C.Canali &C.Jacoboni, F.Nava: Experimental observation of intervalley diffusion in Ge at 77 and 190 K. Supported by Monte Carlo simulations.

- 1979 - Kirkpatrick, Cohen, Dorfman - theory of light scattering from a non-equilibrium fluid.

- 1979 - Gantsevich, Gurevich, Katilius, extended review paper.

- 1979 - Jacoboni and Reggiani: Review of hot electron transport in cubic semiconductors.

- 1979 - Hill, Robson, Fawcett: Diffusion and the power spectral density of velocity fluctuations for electrons in InP by Monte Carlo methods.

- 1979 - R. Barkauskas and R. Katilius: noise with strong ee interactions.

- 1980 - V. Barekeis, V. Viktoravichyus, A. Gal'dikas, and R. Milyushite: 3-band model of GaAs, experiment.

- 1981 - Bareikis et al: experimental measurements of hot hole noise in Ge at liquid He temperatures.

- 1981 - Ernst & Cohen derive hierarchies of coupled equations from the master equation that describe fluctuations.

- 1981 - Jacoboni et al: measurement of drift velocity and diffusivity in Ge vs temperature and field.

- 1982 - Kirkpatrick, Cohen, Dorfman: pedagogical overview of how to treat light scattering from non equilibrium fluids.

- 1982 - Bareikis et al: experimental study of frequency dependence of noise in n-type Si, and InSb (separate paper).

- 1984 - Tremblay: pedagogical overview of theoretical treatment of fluctuations.

- 1987,88 - Bareikis et al: experimental study of effect of source-drain distance on hot electron noise; shorter lengths require higher fields to achieve the same magnitude of noise observed in bulk.

- 1988 - Reggiani & Lugli, Mitin: generalization of Nyquist-Einstein relation for non degenerate semiconductors.

- 1988 - Zimmerman and Yu: MC study of noise in 2DEG.

- 1989 - N.A. Zakhlenyuk, V.A. Kochelap,and V.V. Mitin: Theory of electronic noise with optical phonon emission.

- 1989 - S. Dedulevich, Z. Kancleris, and A. Matulis: noise in weakly heated electron gas.

- 1992 - Bareikis, Katilius, Pozhela, Gantsevich, Gurevich - Fluctuation spectroscopy book chapter.

- 1992 - L. Reggiani T. Kuhn L. Varani: MC simulations of various noise sources in p-type Si at 77 K.

- 1993 - Bareikis et al: real-space transfer noise in 2DEG, experiments.

- 1994 - Nougier: pedagogical overview of noise sources in electronic devices.

- 1994 - Bareikis et al, description of experimental studies on hot electron noise in devices. Effects of intervalley scattering and length dependence are extensively discussed.

- 1994 - Reggiani & Varani: extensive review and MC study of electronic noise in semi-conductors.

- 1994 - Reggiani et al: study of hot phonon effect on electronic noise. Hot phonons are found to increase the noise temperature compared to when equilibrium phonon population is assumed.

- 1995 - Bareikis et al: estimate of Gamma-L intervalley scattering time in GaAs from noise measurements. Value is order 30-50 fs.

- 1997 - Katilius et al: MC study of hot electron noise with ee collisions in GaAs at 80 K.

2.2 Basic concepts [Kogan chap 3]

Applying a voltage across a conductor creates a current but also heats electrons that dissipate the energy to a bath, e.g. thermal lattice vibrations. Heated electrons do not emit the energy instantaneously (e.g. there exists a resistance to energy transfer between electron and phonon systems), and hence they equilibrate at some temperature above the lattice temperature. Current is linear in voltage u, but heating is quadratic, u^2.

Effects that occur at <u>high field</u>: Ohm's law deviations, from mobility decrease and eventually negative differential resistance. Impact ionization generating electron-hole pairs.

Let's define what we mean by 'hot' electrons. If no field is applied, electrons have an equilibrium distribution, call it $f^{eq}(E)$ $(E = energy)$

Fig. 3.1 Equilibrium energy distribution function (curve 1) and the energy distribution function of hot electrons (curve 2). E is the electron energy.
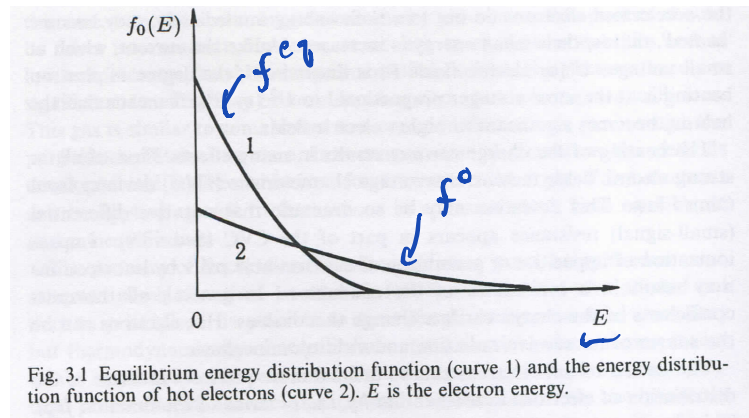
Figure 1:

When an electric field is applied, electrons are heated and come to an equilibrium with occupancy at higher energy states. Therefore, f^0 deviates from f^{eq}. In this situation, we say that we have a 'hot electron gas.'

We will see that hot electron properties depend not only on their mobility, which are determined by the momentum relaxation time τ_1, but also on the energy relaxation time τ_0. These scattering mechanisms can arise from different sources.

$$E = \frac{\hbar^2 k^2}{2m^*}$$

At low temperatures, elastic scattering is due to impurities. Scattering by acoustic phonons is quasi-elastic. For a parabolic band, electrons with momentum $\hbar k$ can change momentum by at most $2\hbar k$. The energy of phonons with that wave vector is $2\hbar s k$, where s is sound speed. Therefore, the ratio

$$\hbar \omega = \hbar s 2k$$

$$s \sim 10^3 \frac{m}{s}$$
$$v \sim 10^5 \, m/s$$

$$\frac{phonon}{electron} = \frac{\hbar \omega}{E(k)} = \frac{2\hbar s k}{\frac{\hbar^2 k^2}{2m^*}} \sim \frac{s}{\hbar k / m^*} = \frac{s}{v} \ll 1$$

where v is electron velocity. Since the Fermi velocity $>>$ sound velocity, momentum can change substantially but energy changes very little. $\rightarrow$ quasi-elastic.

We also consider the various probabilities of phonon emission or absorption since that will set the net balance of energy transferred between electron and phonon systems. Remember that the probability of phonon emission is $\sim N_q + 1$, absorption is N_q. Here the occupancy of phonons with wave vector q is $N_q = \left(e^{\hbar \omega_q / kT} - 1\right)^{-1}$. The ratio of the difference in absorption to emission processes to total is:

$$\frac{\Delta}{total} = \frac{N_q + 1 - N_q}{N_q + 1 + N_q} = \frac{1}{2 N_q + 1}$$

This quantity gives an idea about the net amount of energy that can accumulate in the electron system. If the ratio is small, then electrons can get rid of about as much energy as they gain from the phonons. Taking $\hbar \omega_q \ll k_s T_0$ for acoustic phonons, we see that

$$\frac{\Delta}{total} \sim \frac{1}{N_q} \sim \frac{1}{kT / \hbar \omega_q} \sim \frac{\hbar \omega_q}{k_s T_0} \sim \frac{s}{v} \ll 1$$

That means emission and absorption are almost matched in this limit, and hence the effective rate of energy loss to the phonon thermal bath is reduced compared to the rate of momentum scattering. The reduction therefore scales as $(s/v)^2 \ll 1$.

When we consider impurity scattering, we have $\tau_1 \ll \tau_1^{phon}$ so that the ratio is even bigger: $\sim 10^5$ in n-InSb at 6 K. The result is that for this limit, momentum randomization leads to an isotropic momentum space distribution, even if electrons become hot.

Another useful limit: at low temperatures T_0 and high electron densities n, the electron-electron collision scattering rate, τ_{ee}^{-1}, is greater than energy scattering rate. Then the distribution function of electrons can be approximated by an equilibrium Fermi distribution with temperature T (electron temperature), which is $> T_0$, the lattice temperature.

We can get T by a power balance:

$$\sigma(T) F^2 \simeq P(T)$$

$$\left(F = \text{electric field} \right)$$

LHS: Joule power per mass gained from field

RHS: Power lost to phonon bath

Considering a two-temperature model, we write

$$P(T) \simeq \frac{n C_e k_s}{\tau_e(T)} \cdot (T - T_o)$$

$$\left(\tau_e \longleftrightarrow \tau_o \right)$$

Therefore we find the electron temperature:

$$k_g (T - T_o) = \underbrace{\frac{\sigma F^2 \tau_e}{n C_e}}_{} = \frac{q \mu F^2 \tau_e(T)}{C_e}$$

At sufficiently high fields, high crystal temperatures, electron energy can be comparable to optical phonon energy. In that case, everything discussed above breaks down. We generally need to numerically solve the Boltzmann equation.

2.3 Macroscopic model for hot electron noise in semiconductors

In general obtaining the spectral density for hot electron noise from a microscopic treatment is challenging. We can get some physical insight from a macroscopic model in a simplifying limit; namely high electron density and low temperature. In that case ee collisions are most frequent and inelastic scattering is sufficiently small, so that $f^0(\epsilon)$ is close to an equilibrium one with an electron temperature $T \gg T_o$ lattice temp.

In that case, for frequencies $f \ll \tau_{ee}^{-1} < \tau_I^{-1}$, we can use Langevin sources to write down equations governing the fluctuations of macroscopic properties, current and temperature.

Current Langevin source, due to momentum scattering: $\delta I^{int}(t)$

Energy Langevin source, due to inelastic scattering with phonon thermal bath: $\delta J_e^{int}(t)$

For now, we present the relevant equations as originating phenomenologically. They can be derived from a general theory, assuming ee collisions are frequent. We get

$$\delta I(t) = R^{-1}\delta U + U\left(dR^{-1}/dT\right)\delta T(t) + \delta I^{int}(t)$$

$$V n c_e \frac{d\,\delta T}{dt} = I\,\delta U + U\,\delta I - V\frac{dP}{dT}\delta T + \delta J_e^{int}(t)$$

Here $R(T)$ is resistance of sample, V volume, n electron density, c_e electron heat capacity per electron, $P(T)$ the specific power dissipated by electrons into phonon bath.

$-R^{-1}\delta U$

We see from first equation that current fluctuations are caused by voltage fluctuations as $\curvearrowright \delta T$ well as electron temperature fluctuations (if conductivity depends on electron temperature). Take a Fourier transform, eliminate $\delta T(t)$ from equations to get [HW to fill in details] The small-signal impedance is

$$Z(f) = R_d \frac{1 + j\omega\tau_0\frac{1}{2}(\sigma_d/\sigma + 1)}{1 + j\omega\tau_0\frac{1}{2}(\sigma/\sigma_d + 1)} \tag{1}$$

Small signal conductivity:

$$\sigma_d = \sigma(T)\frac{P' + \sigma'F^2}{P' - \sigma'F^2} \tag{2}$$

with F as electric field, $c_b = \frac{n c_e}{\rho'}$ energy relaxation time, $\sigma = j(F)/F$ is DC con-

$P' = dP/dT$

ductivity. Prime denotes derivative wrt electron temperature.

Total Langevin current source:

$$\delta\tilde{I}^{int} = \left[\frac{U(dR^{-1}/dT)}{VP'(1 - j\omega\tau_0)}\delta J_e^{int} + \delta I^{int}\right]\left[1 - \frac{\sigma'F^2/P'}{1 - j\omega\tau_0}\right]^{-1} \tag{3}$$

$\curvearrowright +?$ $\sim? +$

So, current fluctuations = fluctuations of electron drift velocity, and electron temperature fluctuations, are coupled in high F fields if conductivity depends on electron temperature

F

(as it usually does). If mean current is zero, they are decoupled.

Mechanism of coupling: fluctuation of current at nonzero voltage → fluctuation of Joule power → fluctuation of electron temperature → fluctuation of conductivity → fluctuation of current.

First predicted observable of this effect (Price 1965): dispersion in spectral density of current/voltage noise appears at frequencies τ_ϵ^{-1}, in addition to the dispersion at τ_1^{-1}. This additional contribution of noise was termed 'convective' noise by Price; the contribution associated with momentum relaxation was denoted 'thermal' noise.

We will now see that these noise contributions do not follow the symmetry of the crystal, in the sense that noise perpendicular to and parallel to E field are different even in cubic crystals.

First, we have to figure out the Langevin sources, δI^{int}, δJ_ϵ^{int}.

For frequencies $f \propto \tau_1^{-1}$, current Langevin source δI^{int} has spectral density $4 k_B T R^{-1}$.

Energy flux fluctuation is determined by fluctuations in rates of absorption δJ_q^+ and emission δJ_q^- of phonons of wave vector $\bar{q}$, frequency ω_q :

$$\delta J_\epsilon^{int}(f) = \sum_q \hbar \omega_q \left[\delta J_q^+(t) - \delta J_q^-(t) \right]$$

The assumption is the Markovian one that different scattering events are uncorrelated. Therefore, correlation functions of Langevin flux are nonzero only if fluxes correspond to identical random events. They should also be proportional to mean fluxes as they are a shot-type (Poissonian) process. So,

$\left. \begin{array}{c} both\ + \\ or\ - \end{array} \right\} \rightarrow$ $\left\langle \delta J_{q_1}^{\pm}(t_1)\, \delta J_{q_2}^{\pm}(t_2) \right\rangle = \delta(t_1 - t_2)\, \delta_{q_1, q_2}\, \overline{J_{q_1}^{\pm}}$

$$\left\langle \delta J_{q_1}^+(t_1)\, \delta J_{q_2}^-(t_2) \right\rangle = 0$$

Here $\delta J_{\vec{q}}^{\pm}$ is the mean rate of absorption (upper), emission (lower) of phonons with wave vector $\bar{q}$. Then

$$\langle \delta J_e^{int}(t_1)\, \delta J_e^{int}(t_2)\rangle = \delta(t_1-t_2)\sum_q [\hbar\omega_q]^2 \left(\bar{J}_q^+ + \bar{J}_q^-\right)$$

with spectral density

$$S_{J_e}(f) = 2\sum_q [\hbar\omega]^2 \left(\bar{J}_q^+ + \bar{J}_q^-\right)$$

Given the assumptions of model, e.g. high electron density → high impurity concentration, and low temperatures, momentum relaxation is due to impurities, energy relaxation due to phonons. Thus δI^{int}, δJ_e^{int} are uncorrelated.

We then find that the spectral density of current fluctuations parallel to mean current (with fixed voltage such that $\delta U = 0$) is (after a bit of manipulation):

$$\downarrow \tau_u^{-1}$$

$$S_{\vec{j}\parallel}(f) = S_I^{int} + (1+\omega^2\tau_U^2)^{-1}\left[\frac{(\sigma')^2 F^2}{4k_BT\sigma(T)(P'-\sigma' F^2)^2}\frac{S_{J_e}}{V} + \left(\frac{(P')^2}{(P'-\sigma'^2 F^2)^2}-1\right)\right]S_I^{int} \quad (4)$$

with inverse relaxation time of electron temperature at constant voltage as:

$$\tau_U^{-1} = \frac{P'-\sigma' F^2}{nc_e} \quad (5)$$

Using $\delta \hat{I} = A\delta\vec{j}$, we then get the spectral density of current density fluctuations parallel to field $S_{\vec{j}} = A^2 S_j$; $\cdot S_j^{int} = \frac{S_I^{int}}{A^2} = \frac{4k_BT}{A^2 R} = \frac{4k_B T \sigma A}{A^2 L} = \frac{4k_B T \sigma}{V}$

$$S_{\vec{j}\parallel}(f) = \frac{4k_BT\sigma(T)}{V} \times \qquad (6)$$
$$\left[1+(1+\omega^2\tau_U^2)^{-1}\left(\frac{(\sigma')^2 F^2}{4k_BT\sigma(T)(P'-\sigma' F^2)^2}\frac{S_{J_e}}{V} + \left[\frac{(P')^2}{(P'-\sigma'^2 F^2)^2}-1\right]\right)\right]$$

For small F field, the component of noise $\sim F^2$ is interpreted as resistance noise:

the resistance fluctuates due to energy exchange between electrons and phonons and then from coupling current·current and electron temperature fluctuations.

To get the equation, linearize above result about F^2, then use that from Kirchoff's law,

we know (6)

$$\frac{S_I}{I^2} = \frac{1}{I^2}\frac{4k_bT}{R} = \frac{4k_bTR}{V^2} = \frac{S_V}{V^2}$$

$$S_R = \frac{S_V}{I^2} = \frac{1}{I^2}\cdot\frac{V^2}{I^2}S_I$$

$$= \frac{R^2}{I^2}S_I$$

$$\frac{S_R}{R^2} = \frac{S_I}{I^2}$$

Eq. 6

Therefore, we can write the resistance noise as:

$$\frac{S_I}{I^2} = \frac{S_I}{A_j^2\lambda} = \frac{S_j}{\sigma^2 F^2} = \quad \frac{S_R(f)}{R^2} = \frac{1}{V(1+\omega^2\tau_U^2)}\left[\left(\frac{\sigma'}{\sigma P'}\right)^2\frac{S_{J_e}}{V} + 8k_BT\left(\frac{\sigma'}{\sigma P'}\right)\right] \qquad (7)$$

$$\frac{d\sigma}{dT}$$

Now here is where the anisotropy comes in. Current fluctuations transverse to the mean electric field don't couple electron temperature since current fluctuations transverse to the field don't contribute to Joule power. Therefore, the spectral density perpendicular to field is just the usual Nyquist noise:

$$S_{j\perp}(f) = \frac{4k_aT\,\sigma(T)}{V} = \lim_{\omega\tau_u \gg 1} S_{j\parallel}(f)$$

At high frequencies $\omega\tau_u \gg 1$, the parallel and perp spectral densities coincide.

In the low frequency limit, they are different even in cubic crystals. The sign of the difference depends on the sign of $d\sigma/dT = \sigma'$, or the sign of the nonlinearity of current-voltage characteristic.

1. • If it is superlinear, $\sigma' > 0$, L current noise (parallel to field) is higher than transverse (perp to field).

2. • If it is sublinear, $\sigma' < 0$, L current noise (parallel to field) is lower than
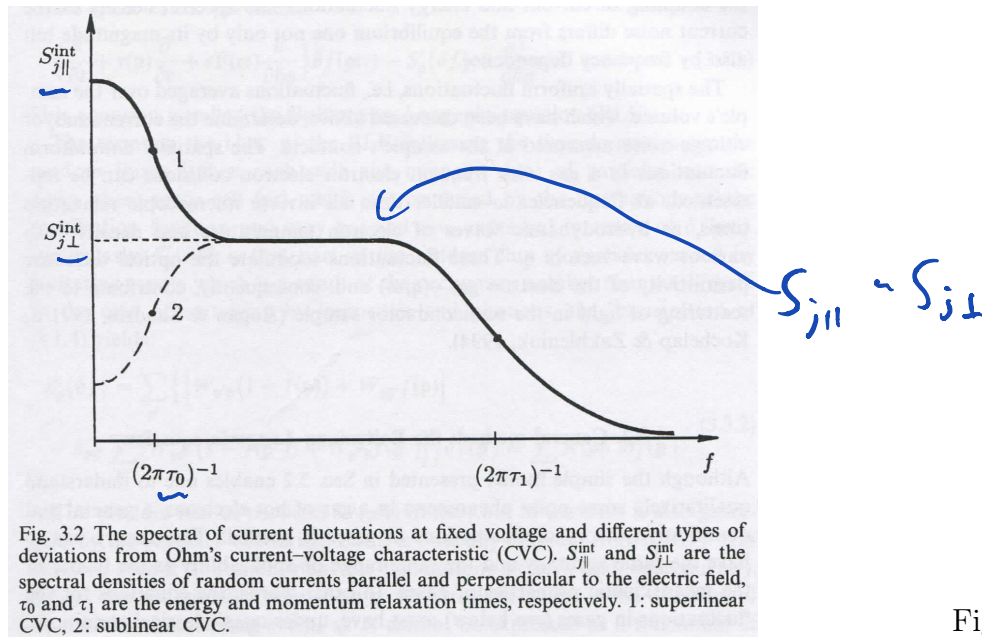
transverse (perp to field).

Fig. 3.2 The spectra of current fluctuations at fixed voltage and different types of deviations from Ohm's current–voltage characteristic (CVC). $S_{j\parallel}^{int}$ and $S_{j\perp}^{int}$ are the spectral densities of random currents parallel and perpendicular to the electric field, τ_0 and τ_1 are the energy and momentum relaxation times, respectively. 1: superlinear CVC, 2: sublinear CVC.

Figure 2:

We can now use the definition of noise temperature as

$$T_N = \frac{S_{\check{\imath}}(f)}{4 k_B \, \text{Re}\left(Z^{-1}(f)\right)}$$

which describes the electrical noise power transmitted from a sample with small-signal impedance $Z(f)$ to a matched load with impedance $Z^*(f)$, in unit frequency band.

We then finally get the equations for longitudinal and transverse noise temperatures:

$$\tau_{\check{\imath}}^{-1} = \frac{P' + \sigma' F^2}{n c_e}$$

$$T_{N\parallel}(f) = T \left[1 + \frac{(\sigma')^2 F^2}{n^2 c_e^2} \left(F^2 + \frac{S_{J_e}}{4VT\sigma} \right) \left[(\tau_I \tau_U)^{-1} + \omega^2 \right]^{-1} \right] \tag{8}$$

At sufficiently high frequencies $\omega \gg \sqrt{\tau_I \tau_u}$, we have $T_N \to T$.

At low frequencies $\omega \ll \sqrt{\tau_I \tau_u}$, longitudinal noise temperature is always higher than the physical temperature. Transverse noise temperature equals physical temperature. (of electrons)

$$T_{N\perp} = T$$

The L spectral density can be bigger or smaller than transverse depending on sign of $d\sigma/dT$. For most semiconductors, $\sigma' < 0$ since scattering rates increase with energy and hence conductivity decreases as higher energy states are occupied (e.g. higher temperature). So we often measure a decrease in spectral noise power with increasing E field for 'warm' electrons.

We should not view this noise as enhanced Nyquist-Johnson noise since the magnitude and frequency dependence of the noise differ from equilibrium one.

Finally, note that what we measure with electrical measurements at contacts on a sample is fluctuations averaged over sample volume = spatially uniform component. There also exist spatially non-uniform fluctuations that can be described as hydrodynamic waves of electron temperature and density of wave vector $\tilde{k}$ [for frequencies $\ll$ all microscopic scattering rates]. They modulate the optical dielectric permittivity $\varepsilon(\tilde{k}, \omega)$ of the electron gas and hence lead to observable light scattering in a semiconductor.